HANDLING DIMENSION SYMBOLS

Speed is often measured by measuring a distance and a time and then dividing the distance by the time (this is average speed):

$$Speed = \frac{distance}{time}$$

If a distance is measured in miles and the time required to travel that distance is measured in hours, the speed will be computed in miles per hour (mi/hr). For example, if a car travels 100 mi in 2 hr, the average speed is

$$\frac{100mi}{2hr} \qquad or \qquad 50\frac{mi}{hr}$$

DIMENSION SYMBOLS

The symbol 100 mi/2 hr looks as if we are dividing 100 mi by 2 hr. It may be argued that we cannot divide 100 mi by 2 hr (we can only divide 100 by 2). Nevertheless, it is convenient to treat dimension symbols such as miles, hours, feet, seconds, and pounds much like numerals and variables, for the reason that correct results can thus be obtained mechanically. Compare, for example,

$$\frac{100x}{2y} = \frac{100}{2} \cdot \frac{x}{y} = 50\frac{x}{y}$$

with

$$\frac{100mi}{2hr} = \frac{100}{2} \cdot \frac{mi}{hr} = 50\frac{mi}{hr}$$

This comparison holds in other situations as shown in the following examples:

Example 1. Compare:

$$3\text{ ft} + 2\text{ ft} = (3+2)\text{ ft} = 5\text{ft}$$

with $3x + 2x = (3+2)x = 5x$

This is similar to using the distributive law.

Example 2. Compare:

$$4\text{in} \times 3\text{in} = (4\times 3)(\text{in}\times\text{in}) = 12\text{in}^2 \text{ (sq in)}$$

with $4x \times 3x = (4\times 3)(x \times x) = 12x^2$

Example 3. Compare:

$$5\text{ men} \times 8\text{ hr} = (5\times 8)(\text{men}\times\text{hr}) = 40\text{ man}\cdot\text{hr}$$

with $5x \times 8y = (5\times 8)(x \times y) = 40xy$

Example 4. Compare:

$$3\text{ dollars/yd} \times 5\text{ yd} = (3 \times 5)\ (\text{dollars/yd} \times \text{yd}) = 15\text{ dollars}$$

with $$3\text{y/x} \times 5\text{x} = (3 \times 5)\ (\text{y/x} \times \text{x}) = 15\text{y}$$

Example 5. Compare:

$$48\text{ in} \cdot \frac{1\text{ft}}{12\text{ in}} = \frac{48}{12}\text{in} \cdot \frac{\text{ft}}{\text{in}} = 4\frac{\text{in}}{\text{in}} \cdot \text{ft} = 4\text{ ft}$$

with $$48\text{x} \cdot \frac{\text{y}}{12\text{x}} = \frac{48}{12} \cdot \text{x} \cdot \frac{\text{y}}{\text{x}} = 4\frac{\text{x}}{\text{x}} \cdot \text{y} = 4\text{y}$$

In each of the examples given, dimension symbols are treated as though they were variables or numerals, and as though a symbol like "3 ft" represented a product of "3" by "ft". A symbol like mi/hr is treated as if it represented a division of miles by hours.

Any two measures can be "multiplied" or "divided." For example,

$$6\text{ ft} \cdot 4\text{ lb} = 24\text{ ft} \cdot \text{lb}$$

$$7\text{ in} \cdot 3\text{ ft} = 21\text{ in} \cdot \text{ft}$$

$$3\text{ ft} \cdot 4\text{ sec} = 12\text{ ft} \cdot \text{sec}$$

$$\frac{8\text{ lb}}{2\text{ ft}} = 4\frac{\text{lb}}{\text{ft}}$$

$$\frac{3\text{ in} \cdot 8\text{ days}}{6\text{ lb}} = 4\frac{\text{in} \cdot \text{days}}{\text{lb}}$$

These "multiplications" and "divisions" may not have sensible interpretations. For example, 2 barns x 4 dances = 8 barn/dances. However, the fact that such amusing examples exist causes us no trouble since they don't come up in practice.

<u>CHANGE OF UNIT</u>

Sometimes a change of unit can be achieved by successive substitutions.

Example 1. Change 25 yd to in.

$$\begin{aligned} 25\text{ yd} &= 25 \times 1\text{ yd} \\ &= 25 \times 3\text{ ft} \qquad (\text{substituting } 3\text{ ft for } 1\text{ yd}) \\ &= 25 \times 3 \times 1\text{ ft} \\ &= 25 \times 3 \times 12\text{ in} \qquad (\text{substituting } 12\text{ in for } 1\text{ ft}) \\ &= 900\text{ in} \end{aligned}$$

The notation of "multiplying by one" can also be used to change units in other situations.

Example 2. Change 7.2 in to yd.

$$7.2 \text{ in} = 7.2 \text{ in} \cdot 1 \cdot 1$$

$$= 7.2 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}}$$

$$= \frac{7.2}{12 \cdot 3} \cdot \frac{\text{in}}{\text{in}} \cdot \frac{\text{ft}}{\text{ft}} \cdot \text{yd}$$

$$= 0.2 \text{ yd}$$

Example 3. Change 60 mi/hr to ft/sec

$$60 \frac{\text{mi}}{\text{hr}} = 60 \frac{\text{mi}}{\text{hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$

$$= \frac{60 \cdot 5280}{60 \cdot 60} \cdot \frac{\text{mi}}{\text{mi}} \cdot \frac{\text{hr}}{\text{hr}} \cdot \frac{\text{min}}{\text{min}} \cdot \frac{\text{ft}}{\text{sec}}$$

$$= 88 \frac{\text{ft}}{\text{sec}}$$

<u>PROBLEMS</u>

1. $36 \text{ ft} \times 1/3 \text{ yd/ft}$

2. $6 \text{ lb} \times 8 \text{ hr/lb}$

3. $3 \text{ ft} \times 2/2 \text{ lb/ft}$

4. $6 \text{ ft} + 2 \text{ ft}$

5. $5 \text{ ft}^2 + 7 \text{ ft}^2$

6. $3/5 \text{ lb/ft} + 7/6 \text{ lb/ft}$

7. $\dfrac{2000 \text{ lb} \times (6 \text{ mi/hr})^2}{100 \text{ ft}}$

Perform the following changes of unit, using substitution or multiplying by one.

8. Change 72 in to ft

9. Change 2 days to sec

10. Change 60 lb/ft to oz/in

11. Change $36/day to cents/hr

12. Change 186,000 mi/sec to mi/yr, (let 365 days = 1 yr)

SOLUTIONS

1. 12 yd
2. 48 hr
3. 3 lb
4. 8 ft
5. 12 ft^2
6. 7/10 $lb^2 ft^2$
7. 720 $(lb - mi^2) / (hr^2 - ft)$
8. 6 ft
9. 172,800 sec
10. 80 oz/in
11. 150 cents/hr
12. 5,865,696,000,000 mi/yr

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