INFINITE SERIES: Popular Series

Geometric Series:

$$a + ar + ar^2 + \ldots + ar^{n-1} + \ldots = \sum_{n=1}^{\infty} ar^{n-1}$$

$a\ and\ r\ are\ fixed\ real\ numbers,\ a \neq 0$

If $|r| < 1$, *the geometric series converges and* $\sum_{n=1}^{\infty} ar^{n-1} = \dfrac{a}{1-r}$.

If $|r| \geq 1$, *the series diverges.*

Telescoping Series:

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{n(n+1)} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

The series converges and its sum is 1.

Factorial Series:

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{1}{n!}$$

The series converges.

Harmonic Series:

$$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n}$$

The series diverges.

P-Series:

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots + \frac{1}{n^p} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

p is a real constant.

If $p > 1$, the p-series converges.
If $p \leq 1$, the p-series diverges.

The Alternating Series Theorem (Leibniz's Theorem):

$$a_1 - a_2 + a_3 - a_4 + ... = \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

The series converges if all three of the following conditions are satisfied:

(1) *The* a_n*'s are all positive*

(2) $a_n \geq a_{n+1}$ *for all* n

(3) $a_n \to 0$

Power Series:

(a) $$c_0 + c_1 x + c_2 x^2 + ... + c_n x^n + ... = \sum_{n=0}^{\infty} c_n x^n$$

center 0, *coefficients* $c_0, c_1, c_2 ...$ constant

(b) $$c_0 + c_1(x-a) + c_2(x-a)^2 + ... + c_n(x-a)^n + ... = \sum_{n=0}^{\infty} c_n (x-a)^n$$

center a, coefficients constant.