INTEGRATION BY PARTIAL FRACTIONS

Using this method of **integration** is an important skill in the working of calculus problems. This involves writing a quotient as a sum of fractions which allows for easier evaluation of the integral in question; for example:

The integral we will solve is: $\int \frac{dx}{1-x^2}$

Note that the denominator can be factored into $1\text{-}x^2 = (1+x)(1\text{-}x)$

Now take the fraction $\frac{1}{(1-x^2)}$ and write it as a sum of fractions.

Write $\frac{1}{(1-x)(1+x)} = \frac{A}{(1+x)} + \frac{B}{(1-x)}$

A & *B* are the values of the constants that we will solve for.

The **cover up method** is one of a number of ways used to solve this.

Solve for *A*:

Take $\frac{1}{(1-x)(1+x)} = \frac{A}{(1+x)} + \frac{B}{(1-x)}$

Multiply by (1+x) on both sides giving

$$\frac{(1+x)}{(1-x)(1+x)} = \frac{A(1+x)}{(1+x)} + \frac{B(1+x)}{(1-x)}$$

Now cancel the (1+x) terms on the rule *A* and in the original fraction which gives

$$\frac{1}{(1-x)} = A + B\frac{(1+x)}{(1-x)}$$

Let x = -1 and substitute

$$\frac{1}{(1-(-1))} = A + B\frac{(1+(-1))}{(1-(-1))}$$

$$\frac{1}{2} = A + B\frac{(0)}{2}$$

From this we get $A = \frac{1}{2}$

Solve for *B*:

$$\frac{1}{(1-x)(1+x)} = \frac{A}{(1+x)} + \frac{B}{(1-x)}$$

Multiply both sides by (1-x)

$$\frac{(1-x)}{(1-x)(1+x)} = \frac{A(1-x)}{(1+x)} + \frac{B(1-x)}{(1-x)}$$

Cancel the (x-1) terms on the *B* and the original fraction.

$$\frac{1}{(1+x)} = \frac{A(1-x)}{(1+x)} + B$$

Set x=1 and substitute

$$\frac{1}{(1+1)} = \frac{A(1-1)}{(1+1)} + B$$

$$\frac{1}{2} = \frac{A(0)}{2} + B$$

From this $B = \frac{1}{2}$

Test your solutions to insure that they equal the original function:

$$\frac{1}{2(1+x)} + \frac{1}{2(1-x)} = \frac{2(1-x)+2(1+x)}{4(1+x)(1-x)} = \frac{2-2x+2+2x}{4(1+x)(1-x)} = \frac{4}{4(1+x)(1-x)} = \frac{1}{(1-x)^2}$$ Solution checks.

Restate the integral as

$$\int \frac{1}{1-x^2}\,\text{dx} = \int \frac{1}{2(1+x)}\,\text{dx} + \int \frac{1}{2(1-x)}\,\text{dx}$$

In the first integral, let u = 1+x, then du = dx; in the second integral, let v = 1-x, then dv = -dx or –dv = dx

We now have

$$\frac{1}{2}\int \frac{1}{u}du - \frac{1}{2}\int \frac{1}{v}dv$$

If we integrate, we get

$$\frac{1}{2}\ln|u| - \frac{1}{2}\ln|v| + c$$

Substitute in the values of u & v

$$\frac{1}{2}\ln|1+x| - \frac{1}{2}\ln|1-x| + c$$

The integral is evaluated.