DERIVATIVES II
Product Rule and Quotient Rule

Product Rule:
The derivative of the product of two differentiable functions *f(x)* and *g(x)* is given by:

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

Quotient Rule:
The derivative of the quotient of two differentiable functions *f(x)* and *g(x)* is given by:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Tips for Remembering:

call *f(x)* $\rightarrow$ Hi
the derivative of Hi called *d*Hi
call *g(x)* $\rightarrow$ Ho
the derivative of Ho called *d*Ho

So the new way is:

Product Rule $\frac{d}{dx}(Hi)(Ho) = (dHi)(Ho) + (Hi)(dHo)$

Quotient Rule $\frac{d}{dx}\left[\frac{(Hi)}{(Ho)}\right] = \frac{(Ho)(dHi) - (Hi)(dHo)}{(Ho)^2}$

Example 1:

$y = (3x^2 - 5x)^5 \cdot \ln(x)$

Let,
Hi = $(3x^2 - 5x)^5$ and Ho = ln(x).

Then,
*d*Hi = $5(3x^2 - 5x)^4$ (6x-5), and *d*Ho = $\left(\frac{1}{x}\right)$

$$\frac{dy}{dx} = 5(3x^2 - 5x)^4 \cdot (6x - 5) \cdot \ln(x) + (3x^2 - 5x)^5 \cdot \left(\frac{1}{x}\right)$$

Example 2:

y = $\dfrac{4x}{3x^2 + 2}$

Let,
Hi = 4x and Ho = $3x^2 + 2$

Then,
*d*Hi = 4 and *d*Ho = 6x

$$\frac{dy}{dx} = \frac{(3x^2 + 2)(4) - (4x)(6x)}{(3x^2 + 2)^2}$$

Example 3:

Let,
y = $\left(2x^4 - 3x^3 + x\right)\left(x^2 - x + 5\right)$

Then,
Hi = $2x^4 - 3x^3 + x$ and Ho = $x^2 - x + 5$

dHi = $8x^3 - 9x^2 + 1$ and dHo = $2x - 1$

$$\frac{dy}{dx} = (8x^3 - 9x^2 + 1)(x^2 - x + 5) + (2x^4 - 3x^3 + x)(2x - 1)$$

Example 4:

Let,
y = $\dfrac{x^2 - 3x + 1}{x^2 - 1}$

Then,
Hi = $x^2 - 3x + 1$ and Ho = $x^2 - 1$

dHi = $2x - 3$ and dHo = $2x$

$$\frac{dy}{dx} = \frac{(x^2 - 1)(2x - 3) - (x^2 - 3x + 1)(2x)}{(x^2 - 1)^2}$$